

Heat conduction in an annulus

This text is integrating part of the homonymous link in [PEEI: a computer program for the numerical solution of systems of partial differential equations](#).

Coordinate system: polar

System of measurement: International System of Units

Coordinates of polar system: $\{\rho, \alpha\}$ of which $[\rho] \equiv [\text{length}]$ $[\alpha] \equiv [\text{plane angle}]$ $\mathbb{R}(\rho) \equiv [0, \infty)$
 $\mathbb{R}(\alpha) \equiv [0, 2\pi)$

Unknown functions: $\tau(\rho, \alpha)$ of which $[\tau] \equiv [\text{temperature}]$

Differential analytic model: $\rho \cdot (\partial^2 \tau(\rho, \alpha) / \partial \rho^2) + \partial \tau(\rho, \alpha) / \partial \rho = 0$

Definition set: $\{\{\rho, \alpha\} / \rho_1 \leq \rho \leq \rho_2\} \quad \rho_1 = 1 \quad \rho_2 = 10$

Case 1-1:

Conditions: $\tau(\rho_1, \alpha) = 1000$ $\tau(\rho_2, \alpha) = 0$ $\partial \tau(\rho, \alpha) / \partial \alpha = 0$

Solution by [1]: $\tau(\rho, \alpha) \equiv (\tau(\rho_1, \alpha) \cdot \ln(\rho_2 / \rho) + \tau(\rho_2, \alpha) \cdot \ln(\rho / \rho_1)) / \ln(\rho_2 / \rho_1)$

Related files: [mad-A.txt](#), [points-1.txt](#), PEEI-mem-1.bin, [cond-1-1.txt](#), [PEEI-sol-1-1.txt](#), [plot-1-1.jpg](#)

Case 1-2:

Conditions: $\tau(\rho_1, \alpha) = 1000$ $\tau(\rho_2, \alpha) = 0$ $\partial \tau(\rho, \alpha) / \partial \alpha = 0$

Solution by [1]: $\tau(\rho, \alpha) \equiv (\tau(\rho_1, \alpha) \cdot \ln(\rho_2 / \rho) + \tau(\rho_2, \alpha) \cdot \ln(\rho / \rho_1)) / \ln(\rho_2 / \rho_1)$

Related files: [mad-A.txt](#), [points-2.txt](#), PEEI-mem-2.bin, [cond-1-2.txt](#), [PEEI-sol-1-2.txt](#), [plot-1-2.jpg](#)

Case 1-3:

Conditions: $\tau(\rho_1, \alpha) = 1000$ $\tau(\rho_2, \alpha) = 0$ $\partial \tau(\rho, \alpha) / \partial \alpha = 0$

Solution by [1]: $\tau(\rho, \alpha) \equiv (\tau(\rho_1, \alpha) \cdot \ln(\rho_2 / \rho) + \tau(\rho_2, \alpha) \cdot \ln(\rho / \rho_1)) / \ln(\rho_2 / \rho_1)$

Related files: [mad-A.txt](#), [points-3.txt](#), PEEI-mem-3.bin, [cond-1-3.txt](#), [PEEI-sol-1-3.txt](#), [plot-1-3.jpg](#)

Case 1-4:

Conditions: $\tau(\rho_1, \alpha) = 1000$ $\tau(\rho_2, \alpha) = 0$ $\partial \tau(\rho, \alpha) / \partial \alpha = 0$

Solution by [1]: $\tau(\rho, \alpha) \equiv (\tau(\rho_1, \alpha) \cdot \ln(\rho_2 / \rho) + \tau(\rho_2, \alpha) \cdot \ln(\rho / \rho_1)) / \ln(\rho_2 / \rho_1)$

Related files: [mad-A.txt](#), [points-4.txt](#), PEEI-mem-4.bin, [cond-1-4.txt](#), [PEEI-sol-1-4.txt](#), [plot-1-4.jpg](#)

Case 1-5:**Conditions:** $T(\rho_1, \alpha) = 1000$ $T(\rho_2, \alpha) = 0$ $\partial T(\rho, \alpha) / \partial \alpha = 0$ **Solution by [1]:** $T(\rho, \alpha) \equiv (T(\rho_1, \alpha) \cdot \ln(\rho_2 / \rho) + T(\rho_2, \alpha) \cdot \ln(\rho / \rho_1)) / \ln(\rho_2 / \rho_1)$ **Related files:** [mad-A.txt](#), [points-5.txt](#), PEEI-mem-5.bin, [cond-1-5.txt](#), [PEEI-sol-1-5.txt](#), [plot-1-5.jpg](#)**Case 2-1:****Conditions:** $T(\rho_1, \alpha) = 0$ $T(\rho_2, \alpha) = 1000$ $\partial T(\rho, \alpha) / \partial \alpha = 0$ **Solution by [1]:** $T(\rho, \alpha) \equiv (T(\rho_1, \alpha) \cdot \ln(\rho_2 / \rho) + T(\rho_2, \alpha) \cdot \ln(\rho / \rho_1)) / \ln(\rho_2 / \rho_1)$ **Related files:** [mad-A.txt](#), [points-1.txt](#), PEEI-mem-1.bin, [cond-2-1.txt](#), [PEEI-sol-2-1.txt](#), [plot-2-1.jpg](#)**Case 2-2:****Conditions:** $T(\rho_1, \alpha) = 0$ $T(\rho_2, \alpha) = 1000$ $\partial T(\rho, \alpha) / \partial \alpha = 0$ **Solution by [1]:** $T(\rho, \alpha) \equiv (T(\rho_1, \alpha) \cdot \ln(\rho_2 / \rho) + T(\rho_2, \alpha) \cdot \ln(\rho / \rho_1)) / \ln(\rho_2 / \rho_1)$ **Related files:** [mad-A.txt](#), [points-2.txt](#), PEEI-mem-2.bin, [cond-2-2.txt](#), [PEEI-sol-2-2.txt](#), [plot-2-2.jpg](#)**Case 2-3:****Conditions:** $T(\rho_1, \alpha) = 0$ $T(\rho_2, \alpha) = 1000$ $\partial T(\rho, \alpha) / \partial \alpha = 0$ **Solution by [1]:** $T(\rho, \alpha) \equiv (T(\rho_1, \alpha) \cdot \ln(\rho_2 / \rho) + T(\rho_2, \alpha) \cdot \ln(\rho / \rho_1)) / \ln(\rho_2 / \rho_1)$ **Related files:** [mad-A.txt](#), [points-3.txt](#), PEEI-mem-3.bin, [cond-2-3.txt](#), [PEEI-sol-2-3.txt](#), [plot-2-3.jpg](#)**Case 2-4:****Conditions:** $T(\rho_1, \alpha) = 0$ $T(\rho_2, \alpha) = 1000$ $\partial T(\rho, \alpha) / \partial \alpha = 0$ **Solution by [1]:** $T(\rho, \alpha) \equiv (T(\rho_1, \alpha) \cdot \ln(\rho_2 / \rho) + T(\rho_2, \alpha) \cdot \ln(\rho / \rho_1)) / \ln(\rho_2 / \rho_1)$ **Related files:** [mad-A.txt](#), [points-4.txt](#), PEEI-mem-4.bin, [cond-2-4.txt](#), [PEEI-sol-2-4.txt](#), [plot-2-4.jpg](#)**Case 2-5:****Conditions:** $T(\rho_1, \alpha) = 0$ $T(\rho_2, \alpha) = 1000$ $\partial T(\rho, \alpha) / \partial \alpha = 0$ **Solution by [1]:** $T(\rho, \alpha) \equiv (T(\rho_1, \alpha) \cdot \ln(\rho_2 / \rho) + T(\rho_2, \alpha) \cdot \ln(\rho / \rho_1)) / \ln(\rho_2 / \rho_1)$ **Related files:** [mad-A.txt](#), [points-5.txt](#), PEEI-mem-5.bin, [cond-2-5.txt](#), [PEEI-sol-2-5.txt](#), [plot-2-5.jpg](#)**Case 3-1:****Conditions:** $T(\rho_1, \alpha) = 1000$ $\partial T(\rho_2, \alpha) / \partial \rho + h \cdot (T(\rho_2, \alpha) - T_\infty) = 0$ $h = 2.5$ $T_\infty = 10$ $\partial T(\rho, \alpha) / \partial \alpha = 0$ **Solution by [1]:** $T(\rho, \alpha) \equiv (T(\rho_1, \alpha) \cdot (1 + h \cdot \rho_2 \cdot \ln(\rho_2 / \rho)) + h \cdot \rho_2 \cdot T_\infty \cdot \ln(\rho / \rho_1)) / (1 + h \cdot \rho_2 \cdot \ln(\rho_2 / \rho_1))$ **Related files:** [mad-B.txt](#), [points-1.txt](#), PEEI-mem-1.bin, [cond-3-1.txt](#), [PEEI-sol-3-1.txt](#), [plot-3-1.jpg](#)**Case 3-2:****Conditions:** $T(\rho_1, \alpha) = 1000$ $\partial T(\rho_2, \alpha) / \partial \rho + h \cdot (T(\rho_2, \alpha) - T_\infty) = 0$ $h = 2.5$ $T_\infty = 10$ $\partial T(\rho, \alpha) / \partial \alpha = 0$ **Solution by [1]:** $T(\rho, \alpha) \equiv (T(\rho_1, \alpha) \cdot (1 + h \cdot \rho_2 \cdot \ln(\rho_2 / \rho)) + h \cdot \rho_2 \cdot T_\infty \cdot \ln(\rho / \rho_1)) / (1 + h \cdot \rho_2 \cdot \ln(\rho_2 / \rho_1))$ **Related files:** [mad-B.txt](#), [points-2.txt](#), PEEI-mem-2.bin, [cond-3-2.txt](#), [PEEI-sol-3-2.txt](#), [plot-3-2.jpg](#)

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Case 4-1:

Conditions: $T(\rho_1, \alpha) = 1000$ $\{Q = -2 \cdot \pi \cdot \rho \cdot K \cdot (\partial T(\rho, \alpha) / \partial \rho); \forall \rho_1 < \rho < \rho_2\}$ $K = 20$ $Q = 10000$ $\partial T(\rho, \alpha) / \partial \alpha = 0$

Solution by [1]: $T(\rho, \alpha) \equiv T(\rho_1, \alpha) - Q \cdot \ln(\rho / \rho_1) / (2 \cdot \pi \cdot K)$

Related files: [mad-A.txt](#), [points-1.txt](#), PEEI-mem-1.bin, [cond-4-1.txt](#), [PEEI-sol-4-1.txt](#), [plot-4-1.jpg](#)

Case 4-2:

Conditions: $T(\rho_1, \alpha) = 1000$ $\{Q = -2 \cdot \pi \cdot \rho \cdot K \cdot (\partial T(\rho, \alpha) / \partial \rho); \forall \rho_1 < \rho < \rho_2\}$ $K = 20$ $Q = 10000$ $\partial T(\rho, \alpha) / \partial \alpha = 0$

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Bibliography:

[1] H. S. CARSLAW, J. C. JAEGER, *Conduction of Heat in Solids*, second edition, Oxford University Press, 1986, London.